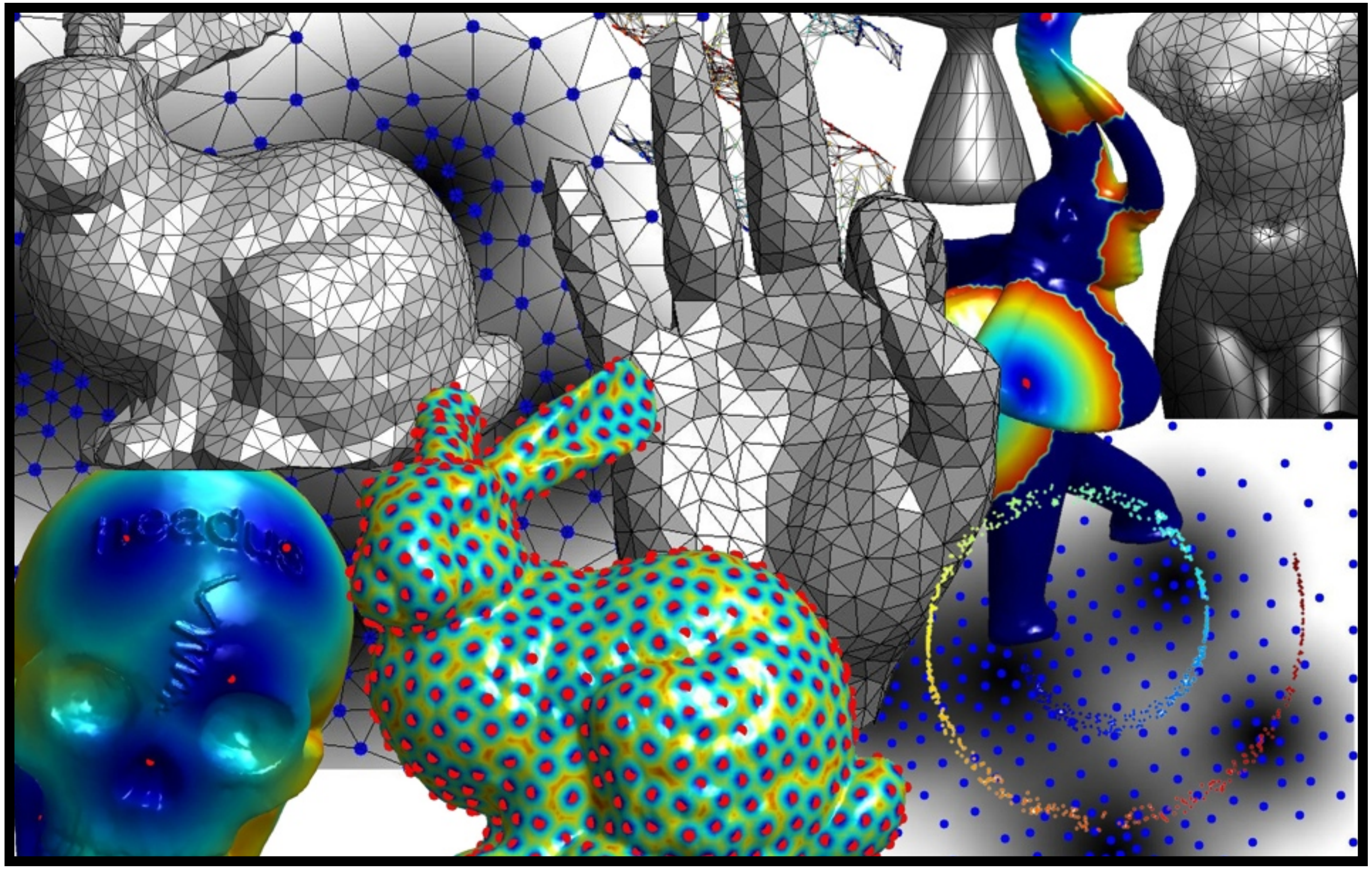
Active Contours



http://www.ceremade.dauphine.fr/~peyre/

Gabriel Peyré

CEREMADE, Université Paris-Dauphine

Overview

- **Parametric Edge-based Active Contours**

- Implicit Edge-based Active Contours

- Area-based Active Contours

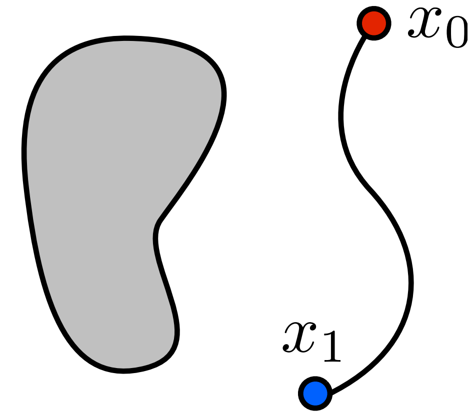
Parametric Active Contours

Local minimum: $$\gamma^\star \in \underset{\gamma}{\operatorname{argmin}}\, E(\gamma) = \underbrace{L(\gamma)}_{\text{Data fidelity}} + \underbrace{\lambda R(\gamma)}_{\text{Regularization}}$$

Boundary conditions:

– Open curve: $\gamma(0) = x_0 \quad \text{and} \quad \gamma(1) = x_1.$

– Closed curve: $\gamma(0) = \gamma(1).$



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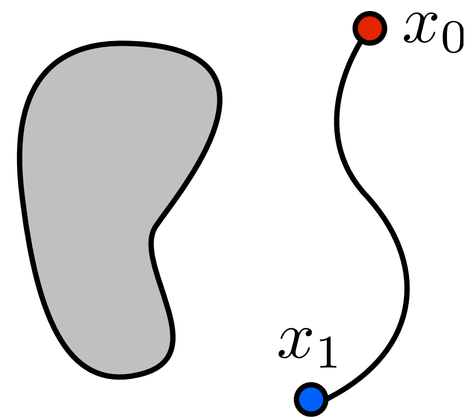


Snakes energy: (depends on parameterization)

$$L(\gamma) = \int_0^1 W(\gamma(t))\|\gamma'(t)\|\mathrm{d}t, \qquad R(\gamma) = \int_0^1 \|\gamma'(t)\| + \mu\|\gamma''(t)\|\mathrm{d}t$$

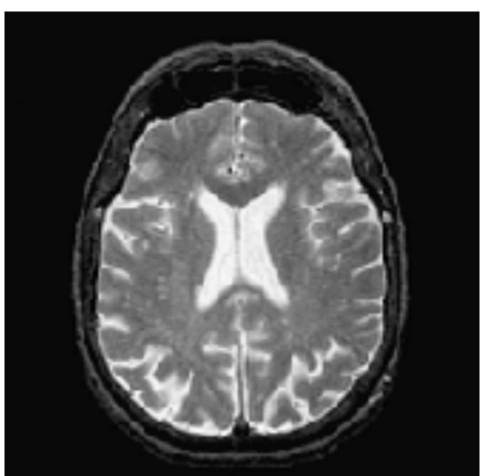
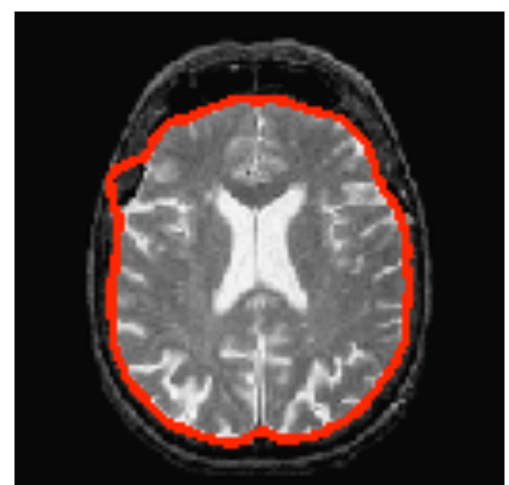


Image f


Weight $W(x)$


Curve $\gamma^\star$

Geodesic Active Contours

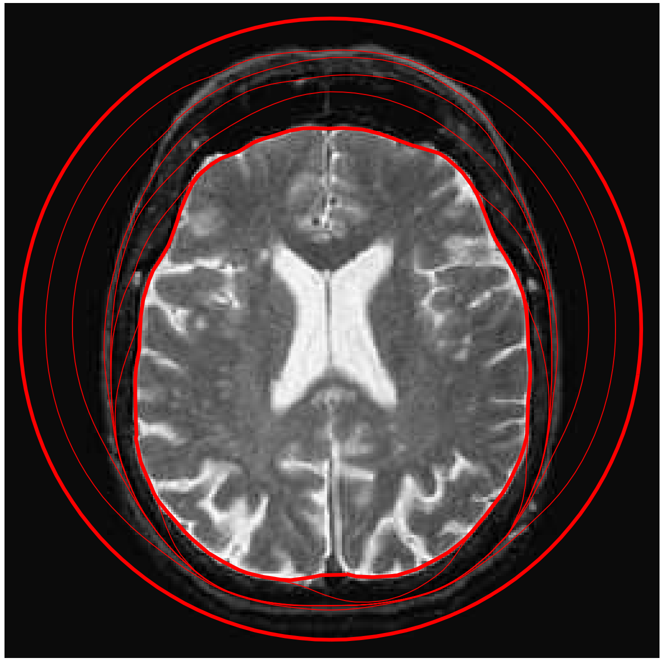
Geodesic active contours: (intrinsic) Replace W by $W + \lambda$,

$$E(\gamma) = L(\gamma) = \int_0^1 W(\gamma(t))\|\gamma'(t)\|\mathrm{d}t$$

$\rightarrow$ (local) minimum of the weighted length L.

$\rightarrow$ local geodesic (not minimal path).

Weight $W(x)$



Curve $\gamma^\star$

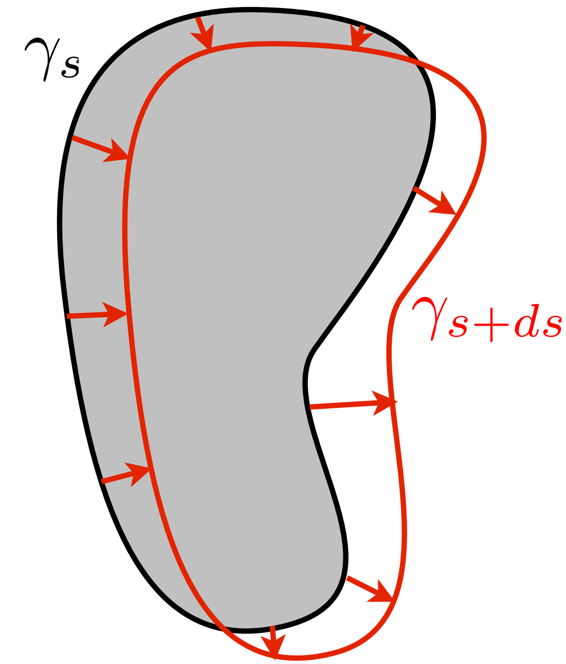
Curve Evolution

Family of curves $\{\gamma_s(t)\}_{s>0}$ minimizing $E(\gamma_s)$.

Do not confound:

t: abscise along the curve.

s: artificial "time" of evolution.



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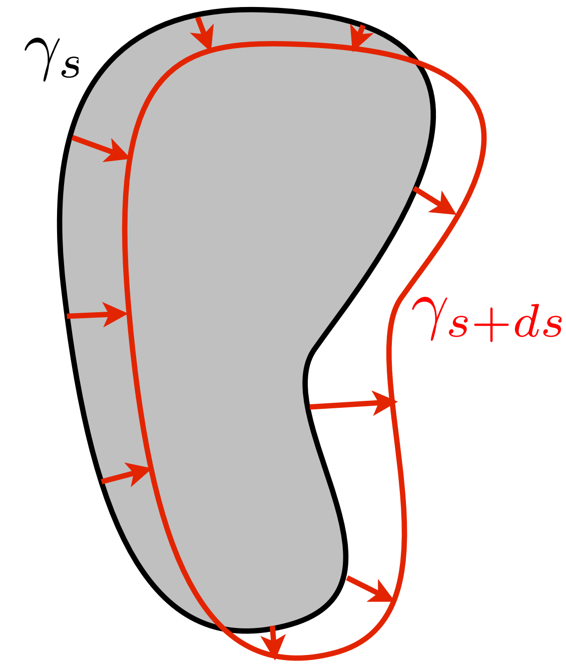


Intrinsic energy E: evolution along the normal

$$\frac{d}{ds}\gamma_s(t) = \underbrace{\beta(\gamma_s(t), n_s(t), \kappa_s(t))}_{\text{speed}} \underbrace{n_s(t)}_{\text{normal}}$$

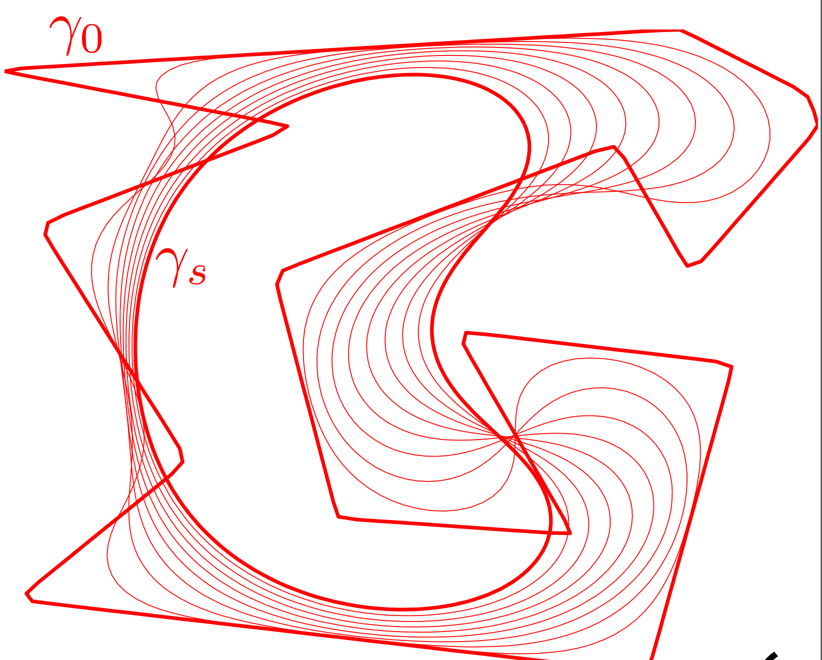
Normal: $$n_s(t) = \frac{\gamma_s'(t)^\perp}{\|\gamma_s'(t)\|}$$

Curvature: $$\kappa_s(t) = \langle n_s'(t),\, \gamma_s'(t)\rangle \frac{1}{\|\gamma_s'(t)\|^2}$$

Mean Curvature Motion

No data-fidelity: $E(\gamma) = \int_0^1 \|\gamma'(t)\| \mathrm{d}t$ Speed: $\beta(x, n, \kappa) = \kappa$

$$\frac{d}{ds}\gamma_s(t) = \kappa_s(t) n_s(t) \quad \longrightarrow \text{Curve-shortening flow.}$$



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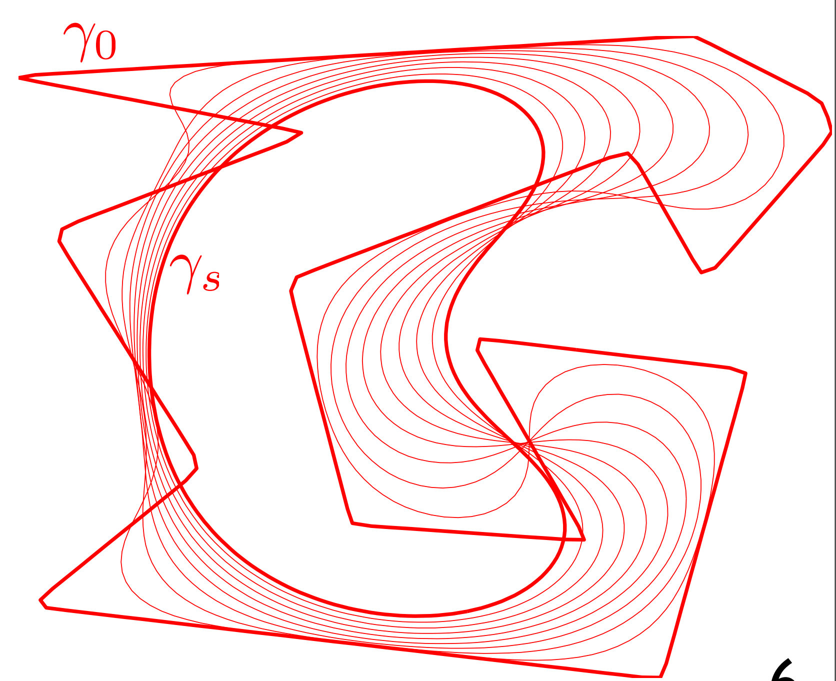
Discretization: $\gamma = \{\gamma(i)\}_{i=0}^{N-1} \subset \mathbb{R}^2$, with $\gamma(N) = \gamma(0)$.

$$E(\gamma) = \sum_i \|\gamma(i) - \gamma(i+1)\|$$

$$\gamma_{k+1}(i) = \gamma_{k+1}(i) - \eta \underbrace{\sum_{i\sim j} \underbrace{\frac{\gamma(i) - \gamma(j)}{\|\gamma(i) - \gamma(j)\|}}_{\text{discretized tangent}}}_{\approx \kappa_s(i/N) n_s(i/N)}$$



Geodesic Active Contours Motion

Weighted length:

$$E(\gamma) = L(\gamma) = \int_0^1 W(\gamma(t)) \|\gamma'(t)\| \mathrm{d}t$$

Evolution:

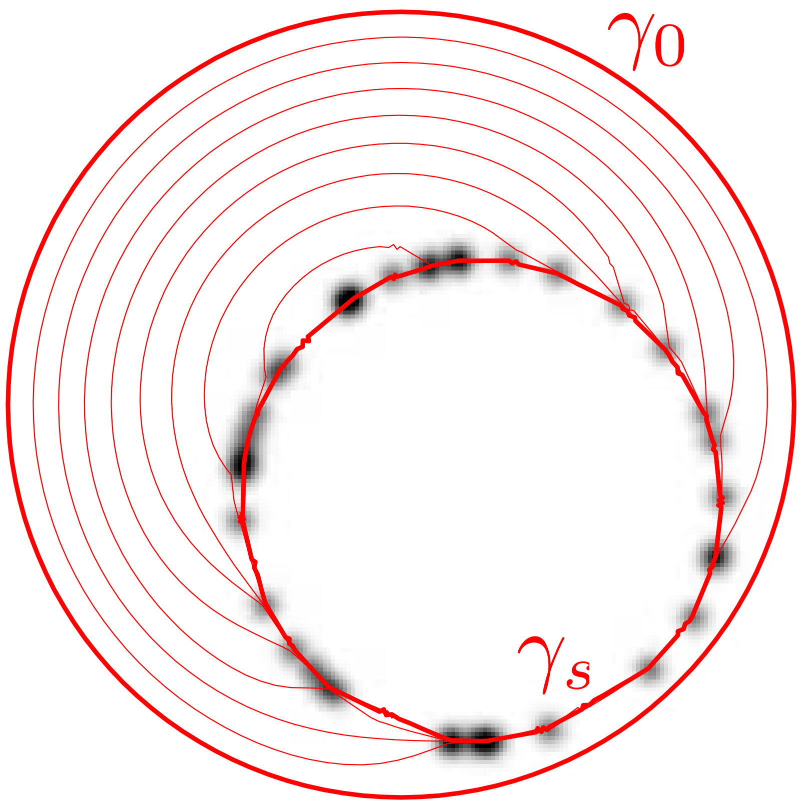
$$\frac{d}{ds}\gamma_s(t) = \beta(\gamma_s(t), n_s(t), \kappa_s(t))\; n_s(t)$$

$$\beta(x, n, \kappa) = W(x)\kappa - \langle \nabla W(x),\, n \rangle$$

$\longrightarrow$ attraction toward areas

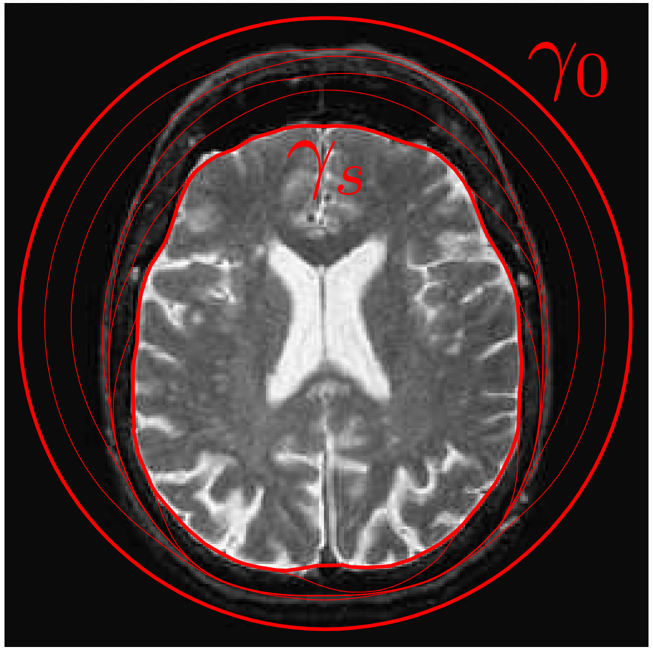
where W is small.

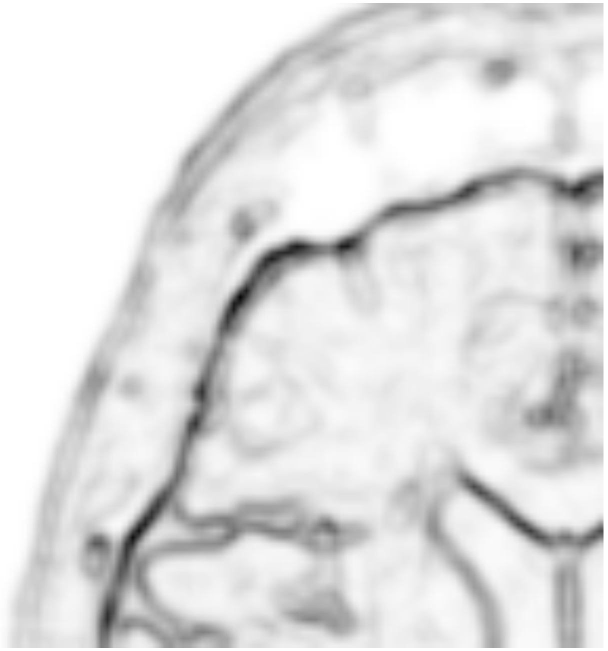
$\longrightarrow$ finite differences discretization.

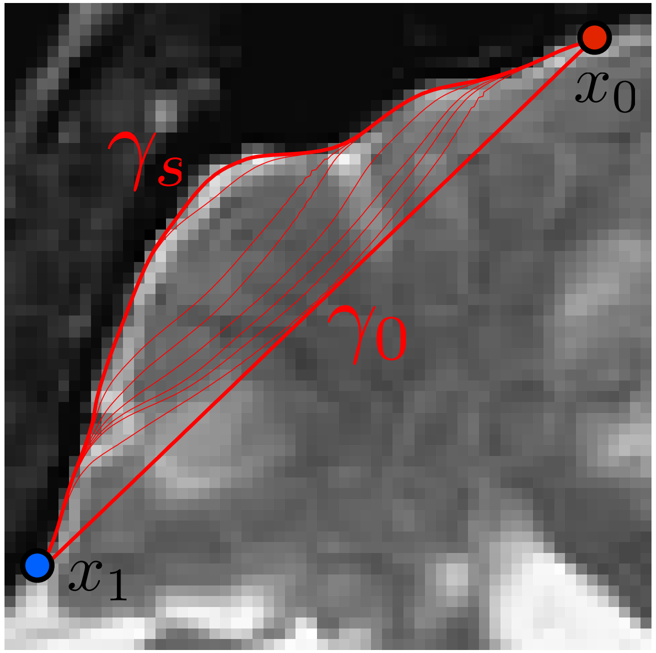


Weight $W(x)$

Open vs. Closed Curves







Weight $W(x)$

Image $f(x)$

Global Minimum with Fast Marching

Geodesic distance map:

$$U_{x_0}(x_1) = \min_{\gamma(0)=x_0, \gamma(1)=x_1} L(\gamma)$$

Global minimum: $U_{x_0}(x_1) = L(\gamma^\star)$

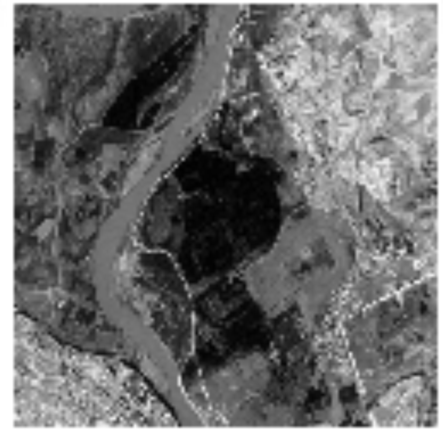
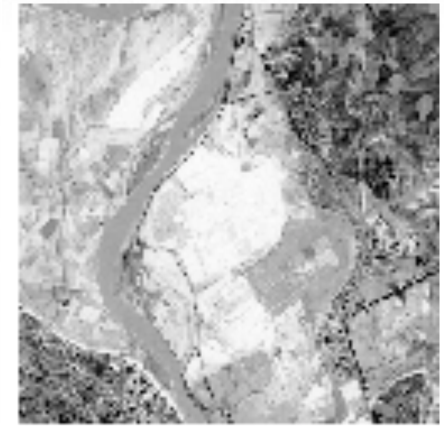
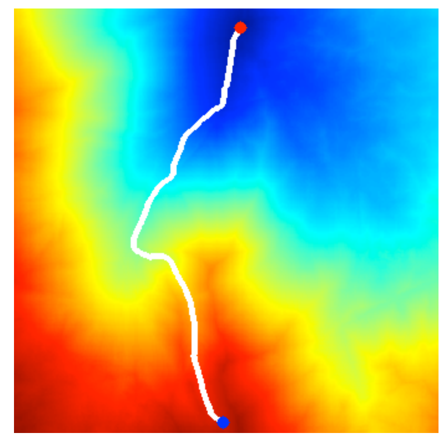


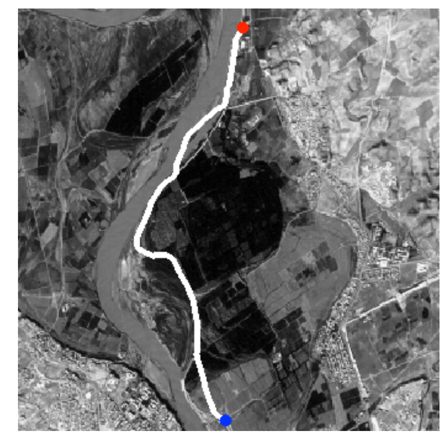
Image f



Metric $W(x)$



Distance $U_{x_0}(x)$



Geodesic curve $\gamma(t)$

Global Minimum with Fast Marching

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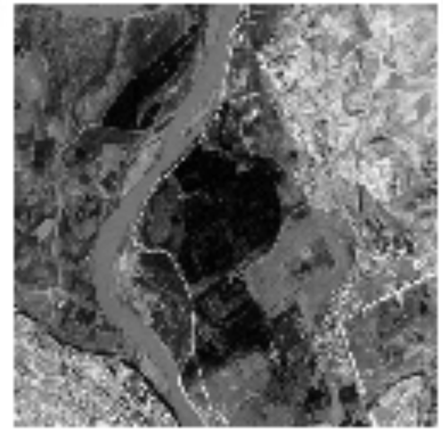
Fast $O(N \log(N))$ algorithm:

– Compute U_{x_0} with Fast Marching.

– Solve EDO:

$$\frac{\mathrm{d}\gamma^\star}{\mathrm{d}t}(t) = -\nabla U_{x_0}(\gamma(t))$$

$$\gamma^\star(0) = x_1$$



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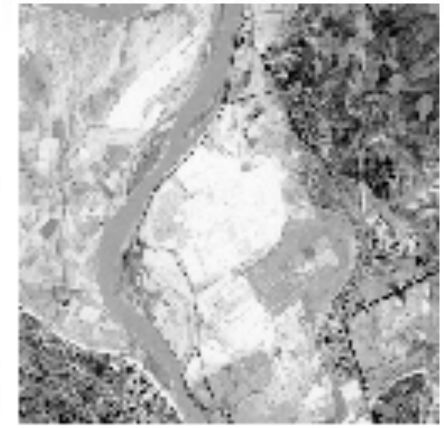

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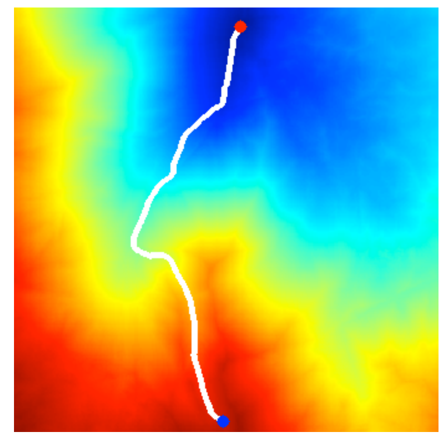

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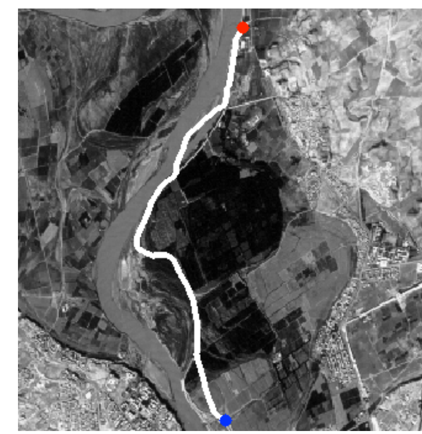
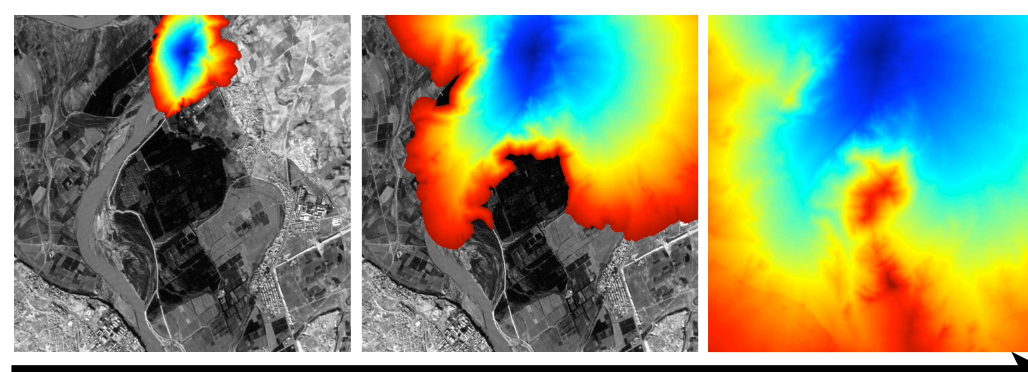


Geodesic curve $\gamma(t)$



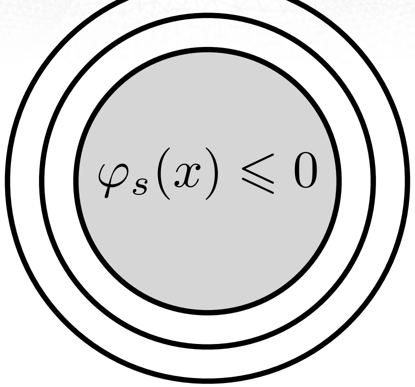
Overview

- Parametric Edge-based Active Contours
- **Implicit Edge-based Active Contours**
- Area-based Active Contor

Level Sets

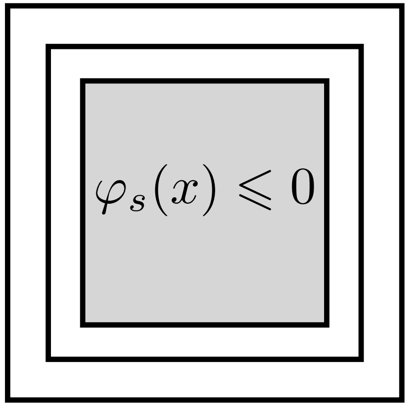
Level-set curve representation:

$$\{\gamma_s(t) \setminus t \in [0,1]\} = \left\{x \in \mathbb{R}^2 \setminus \varphi_s(x) = 0\right\}.$$



Example: circle of radius r $\qquad \varphi_s(x) = \|x - x_0\| - s$

Example: square of radius r $\qquad \varphi_s(x) = \|x - x_0\|_\infty - s$



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Union of domains: $\varphi_s = \min(\varphi_s^1, \varphi_s^2)$

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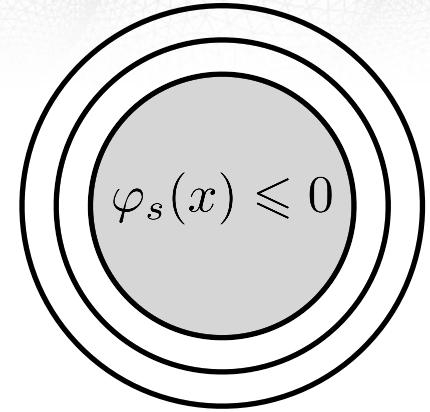

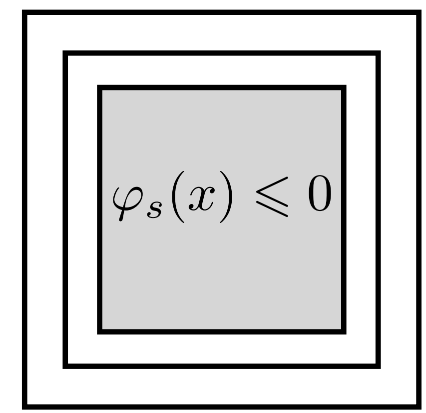

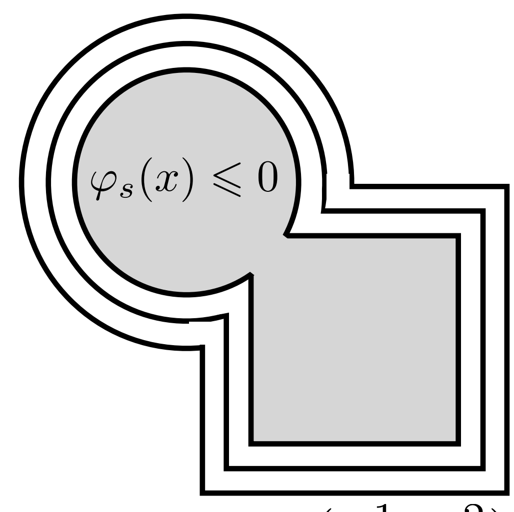

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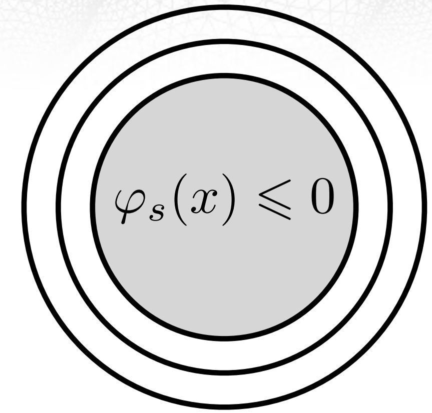

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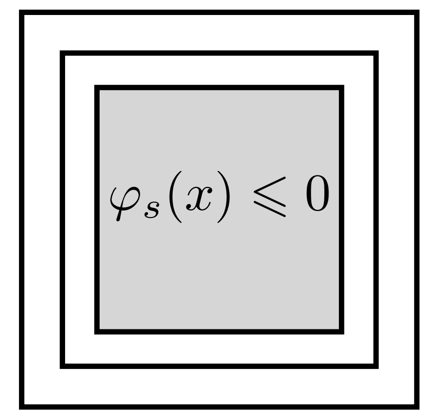


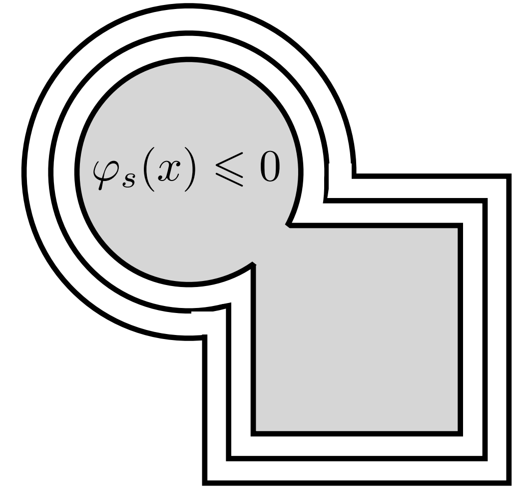
Union of domains: $\varphi_s = \min(\varphi_s^1, \varphi_s^2)$

Intersection of domains: $\varphi_s = \max(\varphi_s^1, \varphi_s^2)$

Popular choice: (signed) distance to a curve

$$\varphi_s(x) = \pm \min_t \|\gamma_s(t) - x\|$$

$\longrightarrow$ infinite number of mappings $\gamma_s \to \varphi_s$.



$\varphi_s = \min(\varphi_s^1, \varphi_s^2)$

Level Sets Evolution

Dictionary parameteric $\leftrightarrow$ implicit:

Position: $x = \gamma_s(t)$

Normal: $n_s(t) = \dfrac{\nabla \varphi_s(x)}{\|\nabla \varphi_s(x)\|}$

Curvature: $\kappa_s(x) = \mathrm{div}\left(\dfrac{\nabla \varphi_s}{\|\nabla \varphi_s\|}\right)(x)$

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Evolution PDE:

$$\frac{d}{ds}\gamma_s(t) = \beta(\gamma_s(t), n_s(t), \kappa_s(t))\, n_s(t)$$

$$\frac{d}{ds}\varphi_s(x) = \|\nabla \varphi_s(x)\| \beta\left(\varphi_s(x), \frac{\nabla \varphi_s(x)}{\|\nabla \varphi_s(x)\|}, \mathrm{div}\left(\frac{\nabla \varphi_s}{\|\nabla \varphi_s\|}\right)(x)\right).$$

$\longrightarrow$ All level sets evolves together.

Implicit Geodesic Active Contours

Evolution PDE:

$$\frac{d}{ds}\varphi_s = \|\nabla \varphi_s\| \mathrm{div}\left(W \frac{\nabla \varphi_s}{\|\nabla \varphi_s\|} \right).$$

Comparison with explicit active contours:

$-$: 2D instead of 1D equation.

$+$: allows topology change.

Implicit Geodesic Active Contours

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Comparison with explicit active contours:

$-$: 2D instead of 1D equation.

$+$: allows topology change.

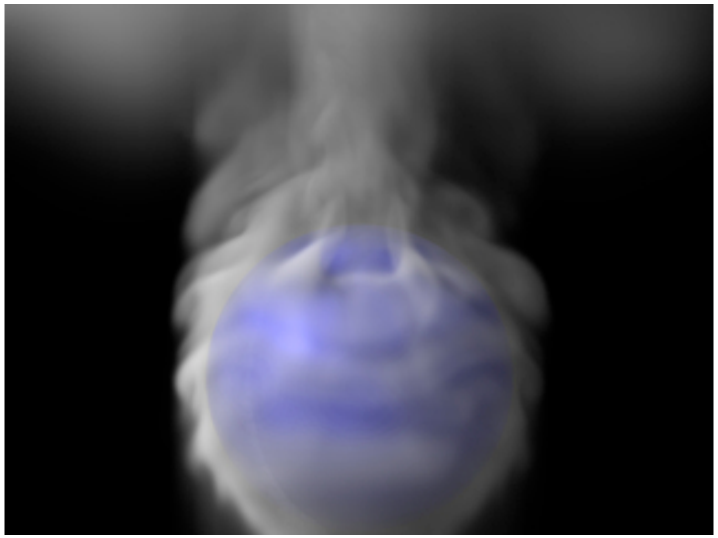
Re-initialization: $\varphi_s(x) = \pm \min_t \|\gamma_s(t) - x\|$

Eikonal equation: $\|\nabla\varphi_s\| = 1$ with $\varphi_s(\gamma_s(t)) = 0$

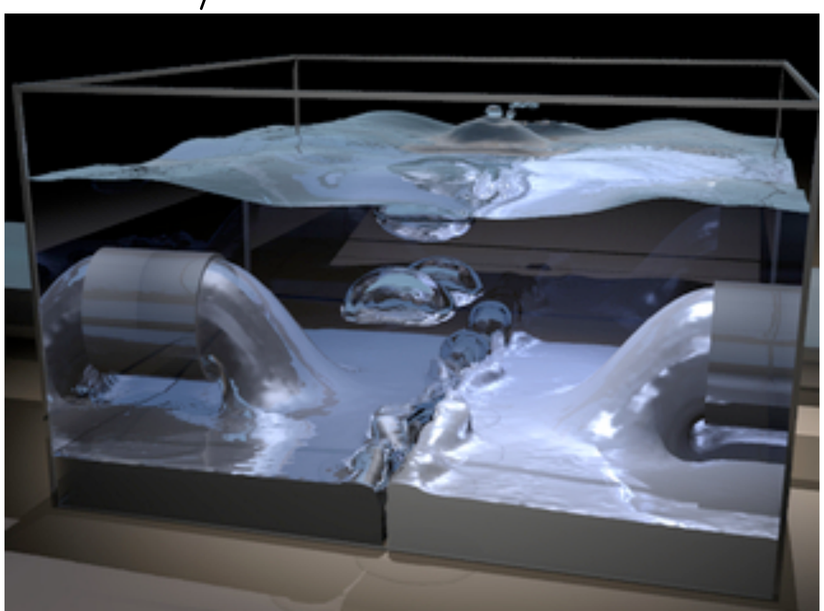
Multiple Fluids Dynamics

See Ron Fedkiw homepage. `http://physbam.stanford.edu/~fedkiw/`

Multiple gaz:




Fluid/air interface:

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- **Area-based Active Contours**